We defined the limit of a sequence as follows.

Definition 1. *The number L is the limit of a sequence $\{a_n\}$, if for any natural number m there exists a number N_m such that $|L - a_n| \leq \frac{1}{m}$ holds for all $n \geq N_m$.*

It is equivalent to the definition following definition given in page 35 of the textbook.

Definition 2. *The number L is the limit of a sequence $\{a_n\}$, if for any positive number $\epsilon > 0$ there exists a number $\bar{N}_\epsilon$ such that $|L - a_n| \leq \epsilon$ holds for all $n \geq \bar{N}_\epsilon$.*

One can observe that ϵ in definition 2 plays the role of $\frac{1}{m}$ in definition 1.

When we show a sequence is unbounded, we will frequently use the following Theorem.

Theorem 3. *Given any number N, there exists a large natural number n such that $n \geq N$ holds.*

Let us consider an example.

Problem 4. *Show that the sequence $\{\sqrt{n}\}_{n \geq 0}$ is not bounded above.*

Proof. Assume that $\{\sqrt{n}\}_{n \geq 0}$ is bounded above. Then, by definition of bounded sequences, there exists a number B such that $\sqrt{n} \leq B$ holds for all $n \geq 0$. Namely, $n \geq B^2$ holds for all $n \geq 0$. This contradicts the above theorem. Therefore, $\{\sqrt{n}\}_{n \geq 0}$ is unbounded. □